Lesson 5: Mathematical Models of Electrical Control System Components

ET 438a

Automatic Control Systems Technology

Learning Objectives

After this presentation you will be able to:

- Identify types of subsystems found in control systems.
- List the characteristics of electrical subsystems.
- Write mathematical models for electrical characteristics.
- Solve for steady-state electrical quantities using given mathematical modeling equations.

Component Models

Subsystem types found in controls systems

- **Electrical** → Motors, Solenoids, Transducers, Control Electronics
- **Mechanical** → Control Valves, Gear Boxes, Linkages
- **Liquid Flow**
- **Gas Flow** → Piping , Tanks, Pumps, Compressors, Filters
- **Thermal** → Heating Elements, Heat Exchangers, Insulation,

Component Models

Systems' behavior defined by component characteristics

Example: electrical components

- Resistance
- Voltage
- Capacitance
- Current
- Inductance
- Charge
- Delay

Component Models

Definition of Electrical Quantities

Resistance	Capacitance	Inductance	Delay
Amount of potential difference required to produce a unit of current.	Amount of charge required to make a unit change in potential.	Amount of potential required to make a unit change in rate of flow (current).	Time interval between signal appearing on input and response appearing on output.

Electrical Component Models

Resistance

Static resistance (linear) $R = \frac{e}{i}$ Ohm's Law

Dynamic Resistance (non-linear) Depends on the values of e and i.

$$R = \frac{\Delta e}{\Delta i} = \frac{de}{di}$$

Can estimate dynamic R with slope of tangent line at operating point.

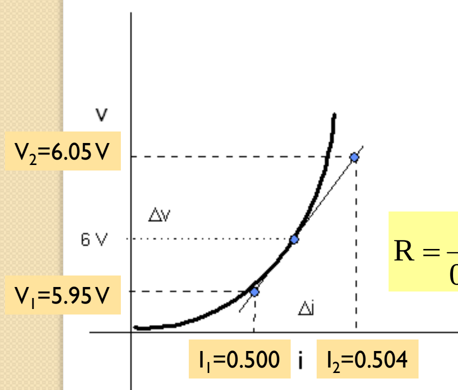
$$R = \frac{\Delta e}{\Delta i} = \frac{e_2 - e_1}{i_2 - i_1}$$

Electrical Component Models

Example 5-1: Non-linear resistance Volt-amp characteristic. Estimate dynamic resistance at the 6V operating point.

$$R = \frac{\Delta V}{\Delta i}$$

$$R = \frac{\Delta e}{\Delta i} = \frac{V_2 - V_1}{i_2 - i_1}$$



$$R = \frac{6.05 - 5.95\ \mathrm{V}}{0.504 - 0.500\ \mathrm{A}} = \frac{0.1\ \mathrm{V}}{0.004\ \mathrm{A}} = 25\ \Omega$$

Electrical Component Models

Capacitance

From the definition of Capacitance $C = \frac{\Delta q}{\Delta e}$

I is rate of change of flow
Coulombs/sec = Amp

Multiple by Δe: $C \cdot \Delta e = \Delta q$

Divide by Δt: $C \cdot \left(\frac{\Delta e}{\Delta t}\right) = \frac{\Delta q}{\Delta t} = i$

Definition of C: $i = C \cdot \left(\frac{de}{dt}\right) = C \cdot \left(\frac{dV_c}{dt}\right)$

Where: V_c =e= voltage across capacitor
C = capacitance in Farads
i = capacitor current in amps

Electrical Component Models

Example 5-2: Sine voltage with amplitude V_m and frequency ω is applied across a capacitor with a value of C Farads. What is the capacitor current?

$$e(t) = V_m \cdot \sin(\omega t)$$

$$i(t) = C \cdot \frac{de}{dt} \qquad \frac{d}{dt}[\sin(x)] = \cos(x)$$

$$i(t) = C \cdot \frac{d}{dt}[V_m \cdot \sin(\omega t)]$$

$$i(t) = C \cdot \omega \cdot V_m \cos(\omega t)$$

90 degree lead between current and voltage. Same as with phasors

Electrical Component Models

Example 5-3: Current pulse of 0.1 sec and amplitude of 0.1 mA is applied to a capacitor. It produces a rise in voltage from 0 to 25 V. What is the capacitance?

Use the incremental definition of C and solve for the value

$$i = C \cdot \left(\frac{\Delta V_c}{\Delta t}\right)$$

$$i \cdot \Delta t = C \cdot \Delta V_c$$

$$\frac{i \cdot \Delta t}{\Delta V_c} = C$$

$V_{c1}=0\text{ V}$ $V_{c2}=25\text{ V}$
$t_1=0$ sec $t_2=0.1$ sec
i=0.1 mA=0.0001 A

$$\Delta V_c = 25 - 0\text{ V} = 25\text{ V}$$

$$\Delta t = 0.1 - 0\text{ sec} = 0.1\text{ sec}$$

$$C = \frac{0.0001 \cdot (0.1)}{25} = 4 \times 10^{-7}\text{ F} = 0.4\ \mu\text{F}$$

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Electrical Component Models

Inductance

$$e = L \cdot \frac{\Delta i}{\Delta t}$$

$$\lim_{\Delta t \to 0} L \cdot \frac{\Delta i}{\Delta t} = L \cdot \frac{di}{dt}$$

$$e = L \cdot \frac{di}{dt}$$

Potential required to make change in current

Dead-time Delay

$$t_d = \frac{D}{v_p}$$

Where: D = distance (m)
v_p = velocity of propagation (m/s)

Use in high frequency transmission lines and satellite communications

Electrical Component Models

Example 5-4: A voltage pulse of amplitude 5 V with a duration of 0.02 seconds is applied across an inductor. This causes a current increase from 1 amp to 2.1 amp. Find L.

Use the incremental definition of L and solve for the value

$$e = L \cdot \left(\frac{\Delta i}{\Delta t}\right)$$

$$e \cdot \Delta t = L \cdot \Delta i$$

$$\frac{e \cdot \Delta t}{\Delta i} = L$$

i_1=1 A i_2=2.1 A
t_1=0 sec t_2=0.02 sec
e=5 V

$$\Delta i_c = 2.1 - 1\text{ A} = 1.1\text{ A}$$

$$\Delta t = 0.02 - 0\text{ sec} = 0.02\text{ sec}$$

$$L = \frac{5 \cdot (0.02)}{1.1} = 0.091\text{ H} = 91\text{ mH} \ \ (\text{V-s/A})$$

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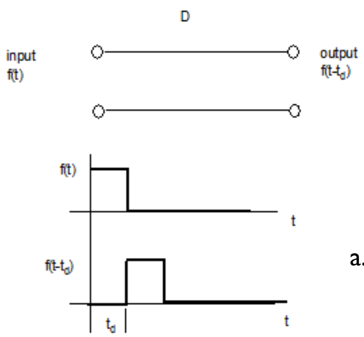
Electrical Delay Examples
Example 5-5

Electrical delays common in long high frequency transmission lines and satellite communications



$$t_d = \frac{D}{v_p}$$

Where v_p = velocity of propagation typical values between 2-3 x10^8 m/s

D = distance (m)

a.) Find the delay of a 600 m transmission line with v_p = 2.3x10^8 m/s

b.) Find the delay of a satellite transmission with a path length of 2000 km and propagation velocity of 3x10^8 m/s.

Electrical Delay Examples

a.) Find the delay of a 600 m transmission line with v_p = 2.3x10^8 m/s

$$t_d = \frac{D}{v_p}$$
$$D = 600\ \text{m}$$
$$v_p = 2.3\times10^8\ \text{m/sec}$$

$$t_d = \frac{600\ \text{m}}{2.3\times10^8\ \text{m/sec}} = 2.609\times10^{-6}\ \text{sec}$$
$$t_d = 2.609\ \mu\text{S}$$

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b.) Find the delay of a satellite transmission with a path length of 2000 km and propagation velocity of 3x10^8 m/s.

Convert km to m

$$D = (2000\ \text{km})\cdot\left(\frac{1000\ \text{m}}{1\ \text{km}}\right) = 2.0\times10^6\ \text{m}$$
$$t_d = \frac{D}{v_p} \quad v_p = 3.0\times10^8\ \text{m/sec}$$

$$t_d = \frac{2.0\times10^6\ \text{m}}{3.0\times10^8\ \text{m/sec}} = 6.667\times10^{-3}\ \text{sec}$$
$$t_d = 6.667\ \text{mS}$$

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END LESSON 5: MATHEMATICAL MODELS OF ELECTRICAL COMPONENTS

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